Course Match: A Large-Scale Implementation of Approximate Competitive Equilibrium from Equal Incomes for Combinatorial Allocation

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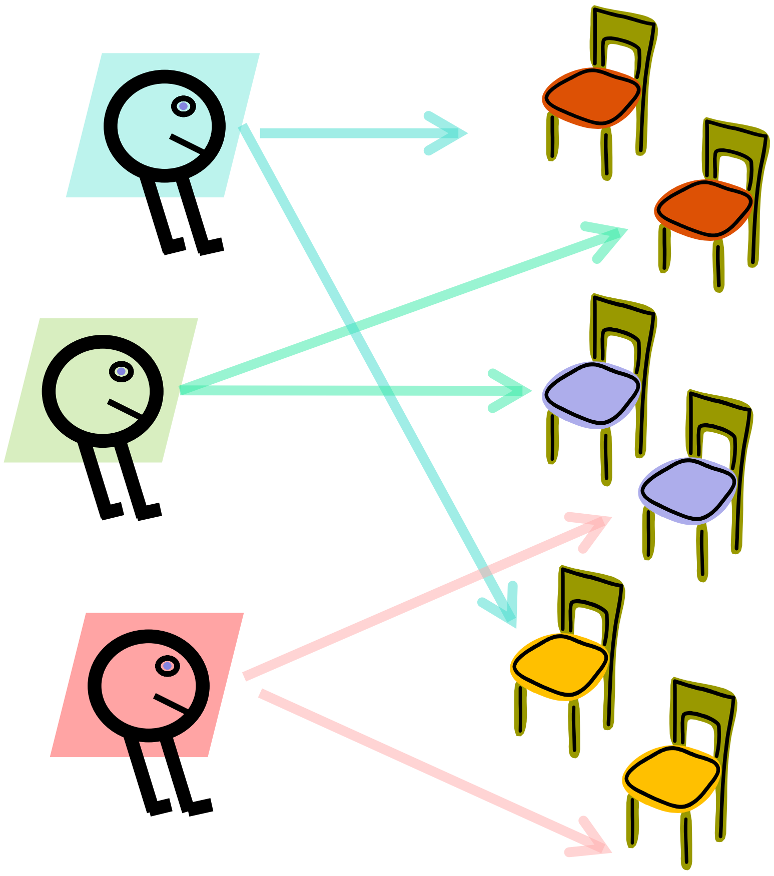
Agenda

- Problem and possible mechanisms
  - Auctions and Course Match
- Theory
  - Combinatorial allocation applied to student assignment to courses
- Implementation
  - The Course Match solution
- Results
  - How did Course Match actually do?

Combinatorial assignment of courses to students

- The problem:
  - Allocate students to course seats
  - ~ 1700 students
  - ~ 350 courses
  - ~ 10,000 seats
- The goal:
  - Maximize student satisfaction
    - Maximize efficiency
    - Maximize fairness
  - Satisfy capacity and scheduling constraints



One solution – an auction

- Problems:
  - Strategic behavior
  - Complexity
  - Pareto inefficient allocations
  - Unfair allocations
  - Low student reported satisfaction

Another solution

COURSE MATCH

Your personal scheduling agent

- Before the semester starts …
  1. Students report preferences
  2. Prices and course schedules are found so that…
     - Each student receives the best schedule (according to their reported preferences) given their budget.
     - Course capacity constraints are not violated.

- After the semester starts …
  - A drop/add period with a FCFS waitlist allows students to modify their schedule.

Additional Course Match details

- Budgets:
  - 2nd year students are given more tokens than 1st year students
  - Unused tokens do not carry over from one semester to another

- Course capacity:
  - *Target capacity* = the desired maximum number of students
  - *Max capacity* = the absolute maximum number of students that can be assigned to a course for a feasible solution
  - Target capacity <= Max capacity

Theory – the bad news

- Serial dictatorship is …
  - The only Pareto efficient and strategy-proof mechanism
  - Terrible with fairness

- A market in which all participants have equal incomes might not have a price equilibrium.

Theory – the good news

- Budish (2011):
  - Participants are given *approximately* equal income
  - The market can be cleared *approximately*
    - There may be oversubscription or undersubscription … but clearing error is bounded
  - Desirable fairness properties can be achieved
  - The mechanism is strategy-proof "in the large"

- Othman, Sandholm, Budish (2010):
  - A Tabu-search method for finding a solution
  - Provide results for small, and simulated problems

Practice – two implementation challenges

- Can students correctly report their preferences?
  - Allow students to report a relative utility for each course and an adjustment for pairs of courses
  - Report to students their “top N” schedules
- Can a good, feasible solution be found?
  - Good if minimizes sum of squared clearing error
  - Good if minimizes deadweight loss
  - Feasible if no violation of the maximum capacity constraint

Finding a good, feasible solution with three stages

- Stage 1: Find a good solution
- Stage 2: Eliminate oversubscription
- Stage 3: Reduce undersubscription

Stage 1: Find a good solution

- Tabu search for a price vector and assignment with less clearing error than the bound.
  - Similar to Othman, Sandholm, Budish (2011).

- Most computationally intensive stage.

Stage 2: Eliminate oversubscription

- Iteratively raise prices so as to reduce by 50% the most oversubscribed course's oversubscription (as measured by maximum capacity).

- Alternatives considered and not adopted:
  - Randomly drop students
  - Lower target capacities
  - Weighted search

Stage 3: Reduce undersubscription

- Increase all budgets by 10%
- Given the stage 2 price vector, iteratively allow students to purchase open seats
  - This increases inequity to reduce deadweight loss
- Alternative not adopted:
  - Let students acquire open seats in the drop/add market

Problem description

Spring 2014	
# of students	1,700
# of courses	344
# of seats	12,523
# of seats assigned	9,316

Computational effort

Spring 2014	
# of compute servers	7
Computing time	48 hours
Number of search starts	418
Search starts performing a strict hill climb	20
Price vectors explored	20 million
Total # of MIPs solved	4.5 billion

Stage 1 results

Compute server	Squared error (over)	Squared error (under)	Seats	Loss
1	51	40	32	0.19%
2	23	28	24	0.20%
3	75	35	25	0.31%
4	36	125	71	0.68%
5	59	48	36	0.28%
6	53	48	32	0.22%
7	47	56	42	0.54%

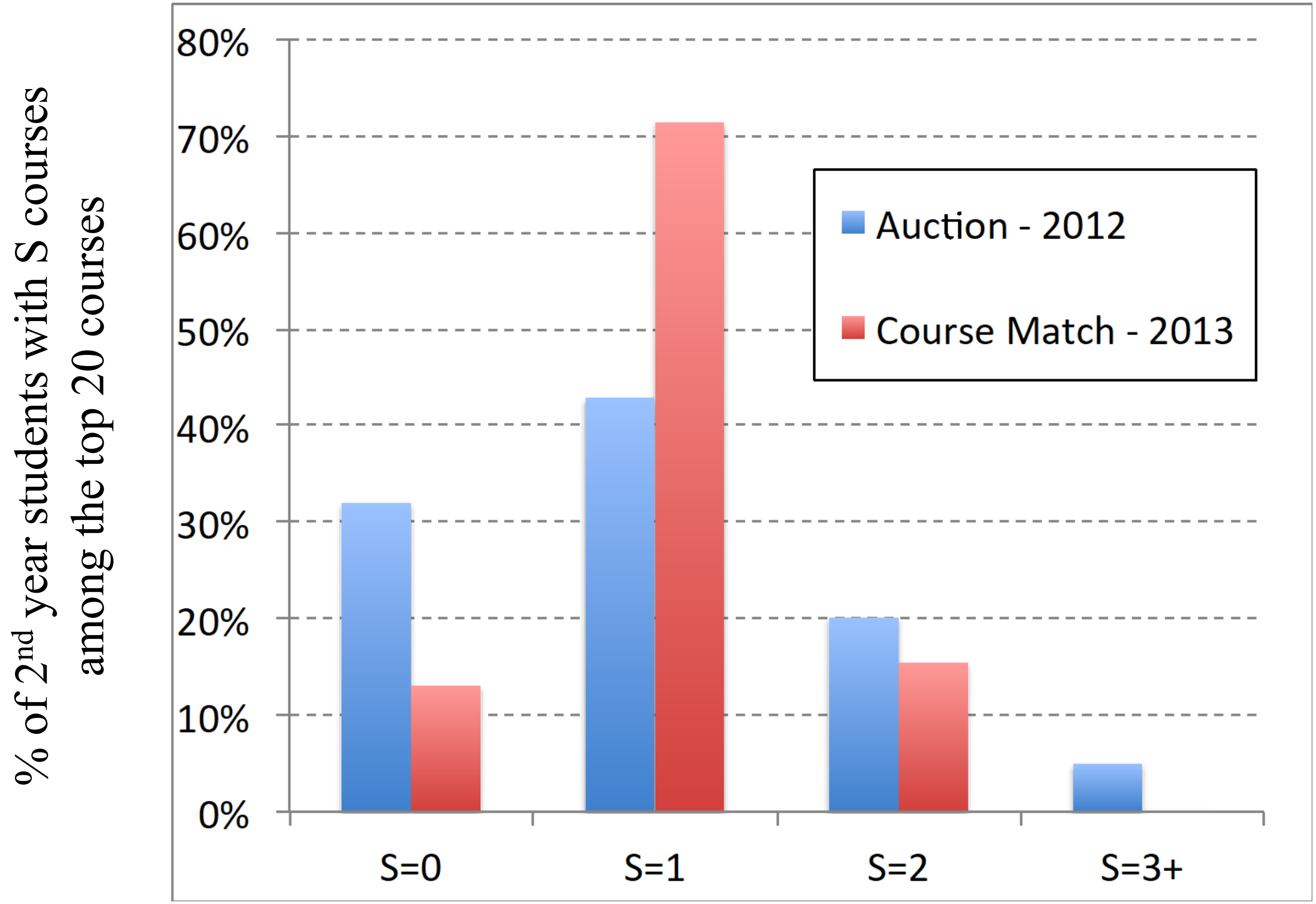
Stage 2 results

Compute server	Squared error (over)	Squared error (under)	Seats	Loss
1	0	141	67	0.80%
2	0	205	97	1.26%
3	0	138	74	0.96%
4	0	228	106	1.00%
5	0	201	87	1.15%
6	0	186	86	0.87%
7	0	202	86	1.13%

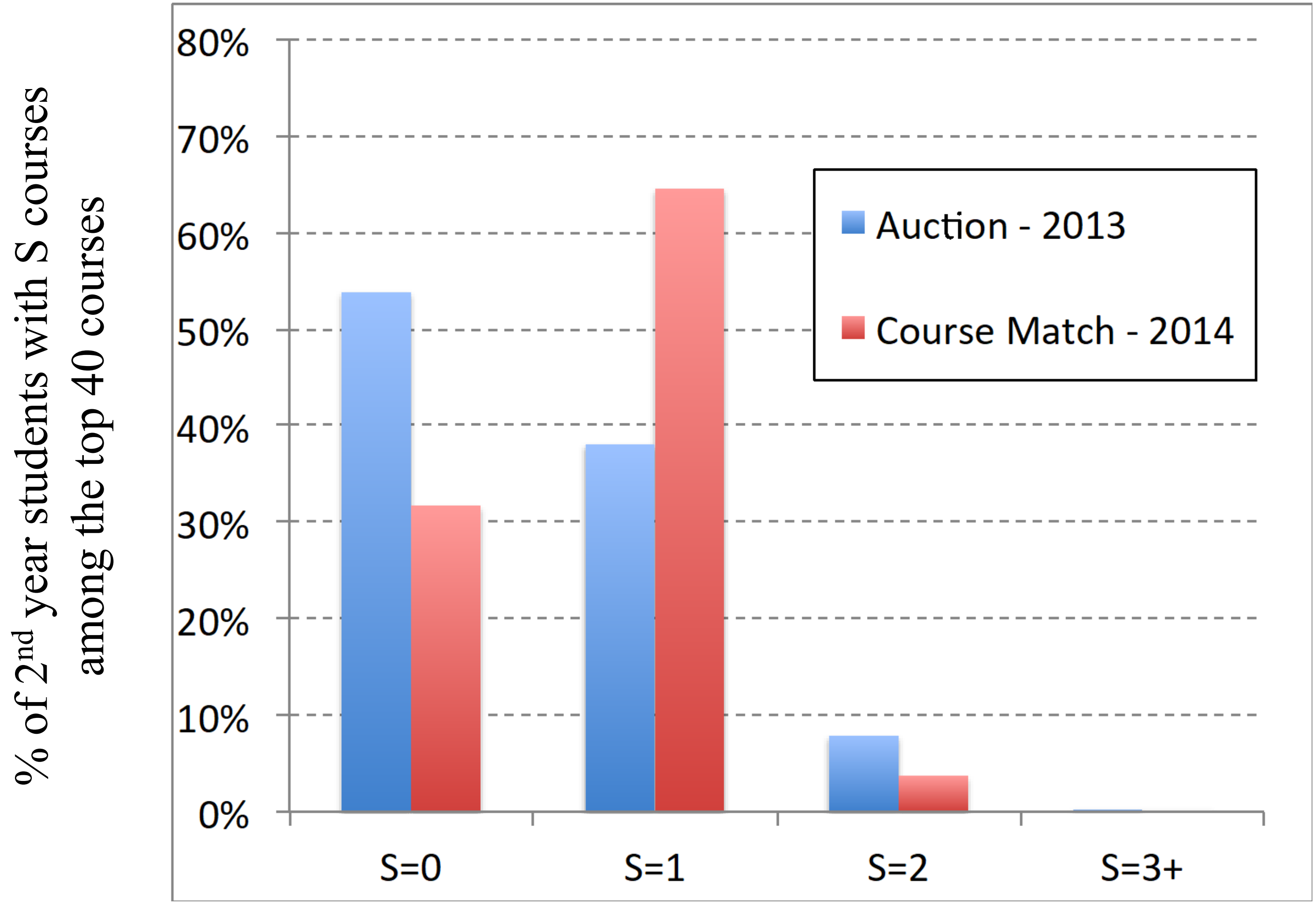
Stage 3 results

Compute server	Squared error (over)	Squared error (under)	Seats	Loss
1	0	31	17	0.07%
2	0	30	16	0.07%
3	0	21	17	0.07%
4	0	83	33	0.33%
5	0	52	24	0.09%
6	0	32	22	0.07%
7	0	17	13	0.02%

Access to top 20 courses in the Fall semester



Access to top 40 courses in the Fall semester



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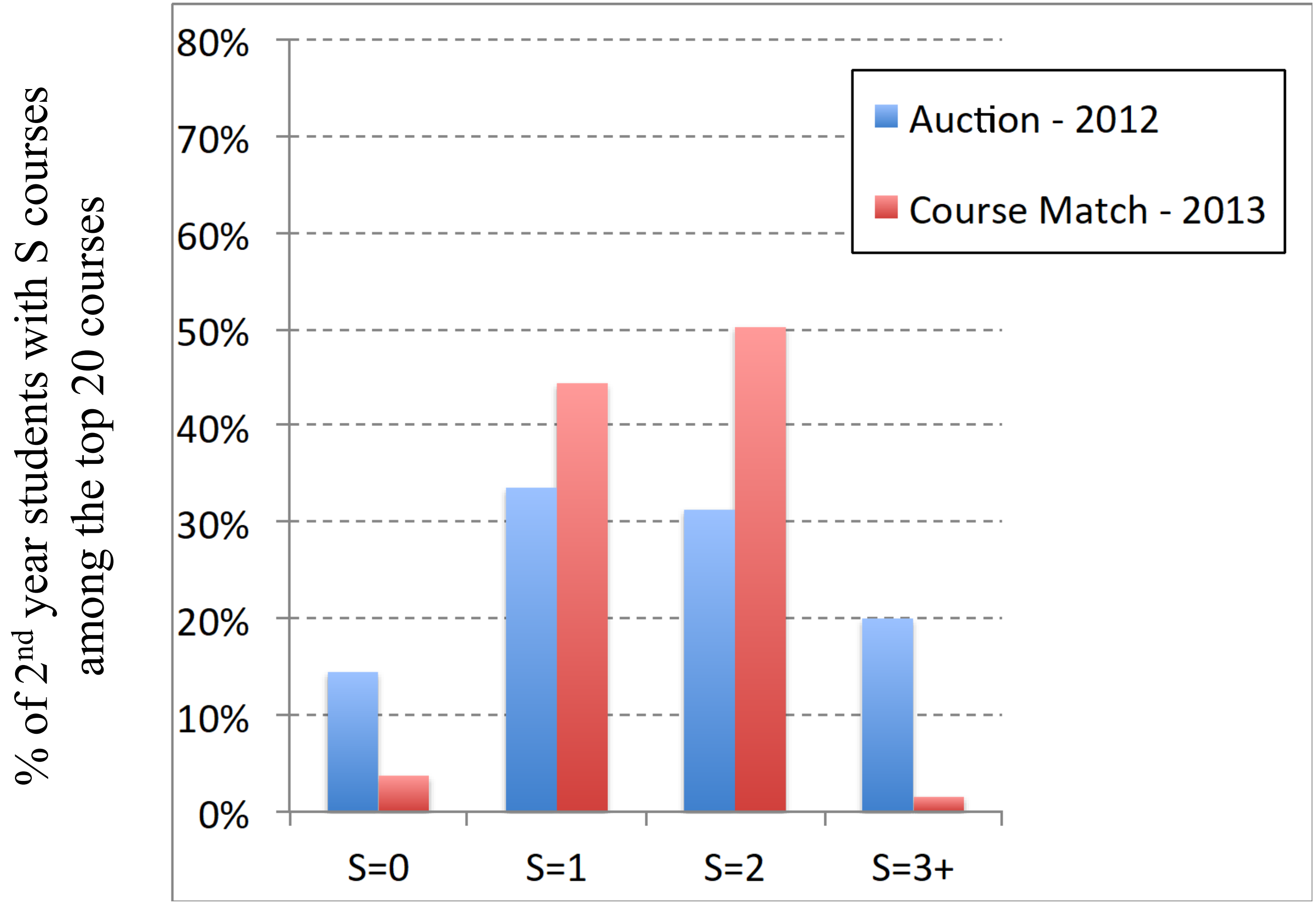

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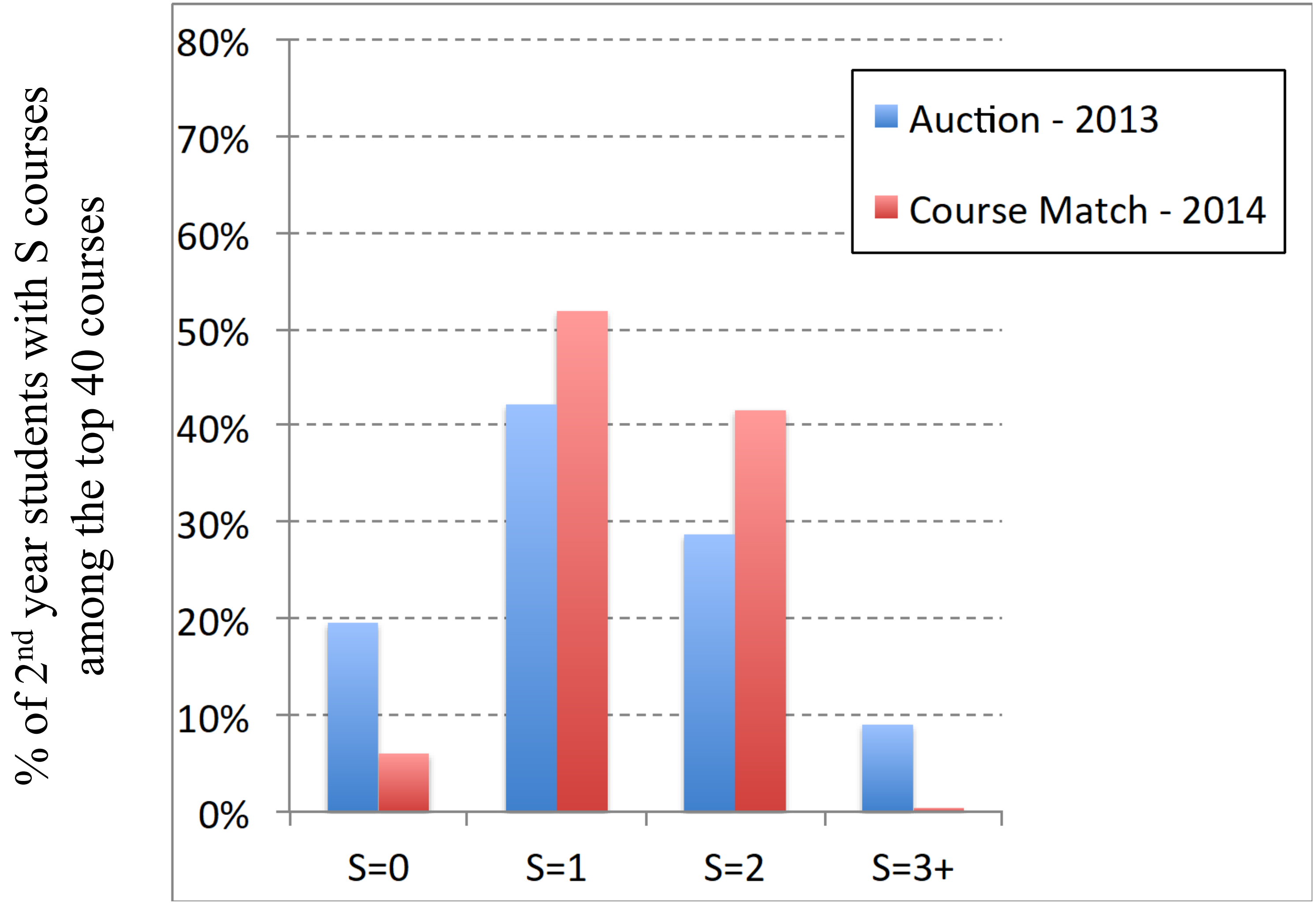
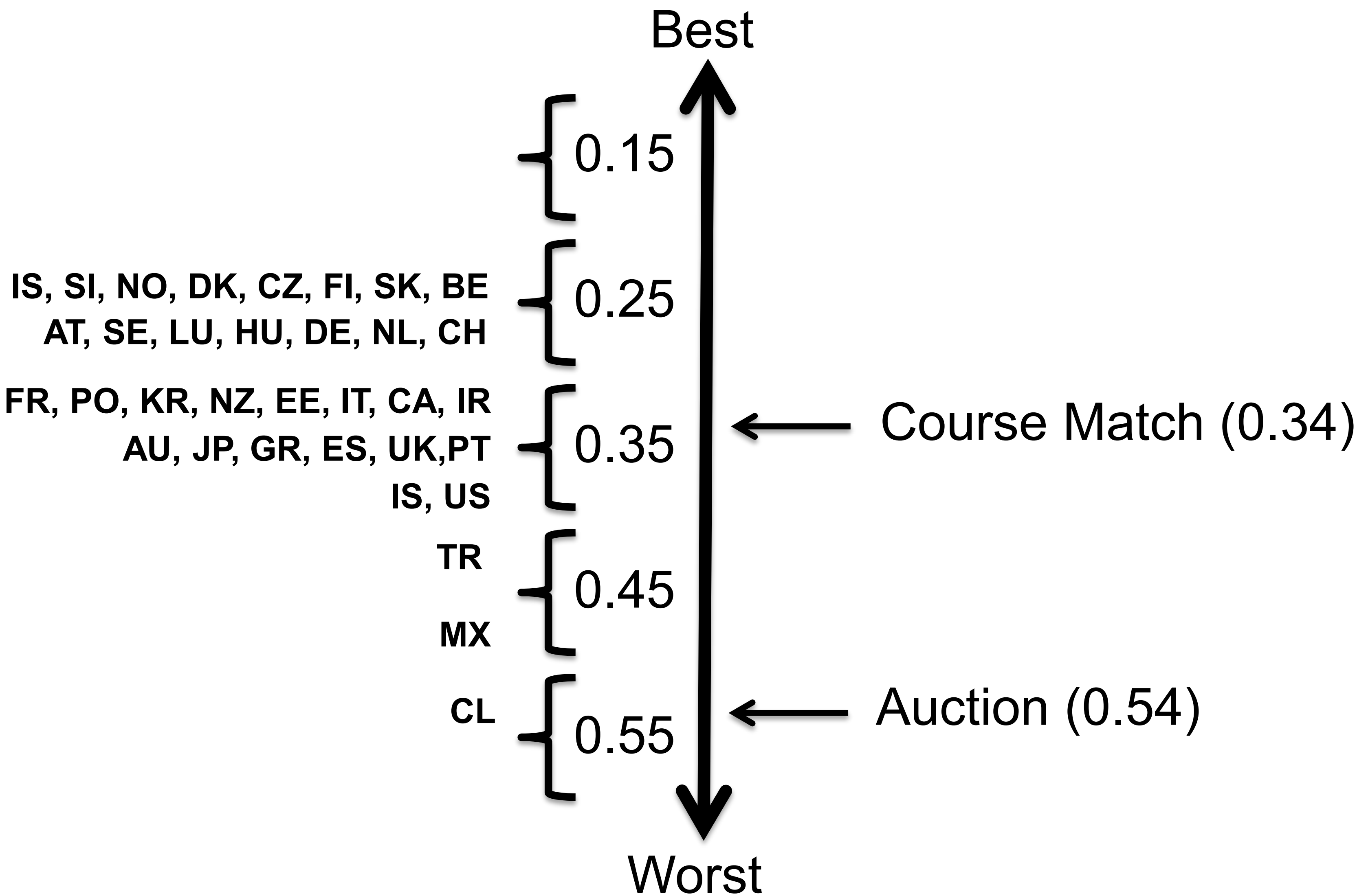


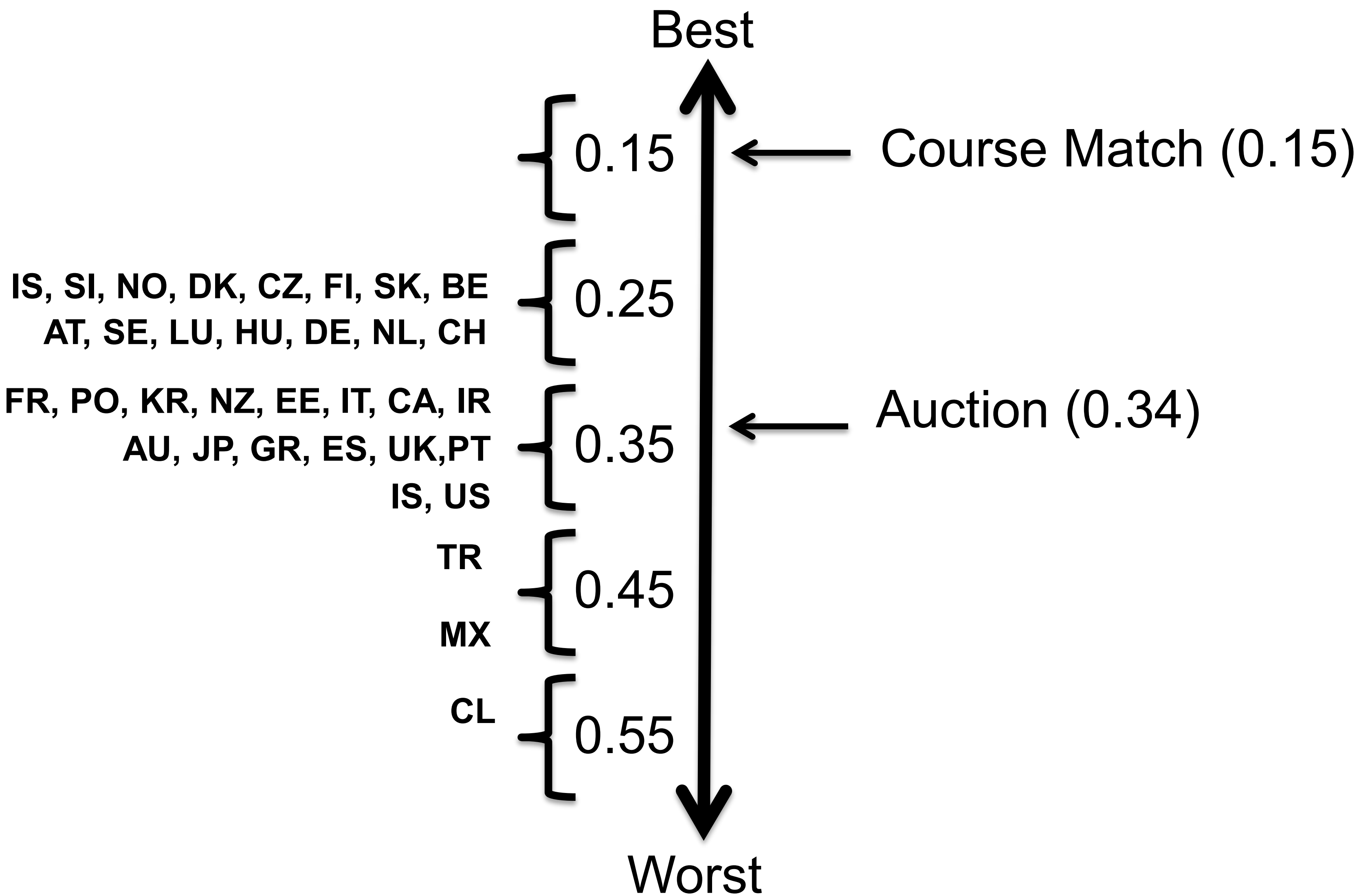
Fairness measured by the Gini coefficient

Mechanism	Semester	1st year students	2nd year students	All students
Auction	Fall 2012	0.33	0.36	0.54
Course Match	Fall 2013	0.13	0.22	0.34
Auction	Spring 2012	0.25	0.39	0.34
Course Match	Spring 2014	0.10	0.12	0.15

Fairness – Fall semester

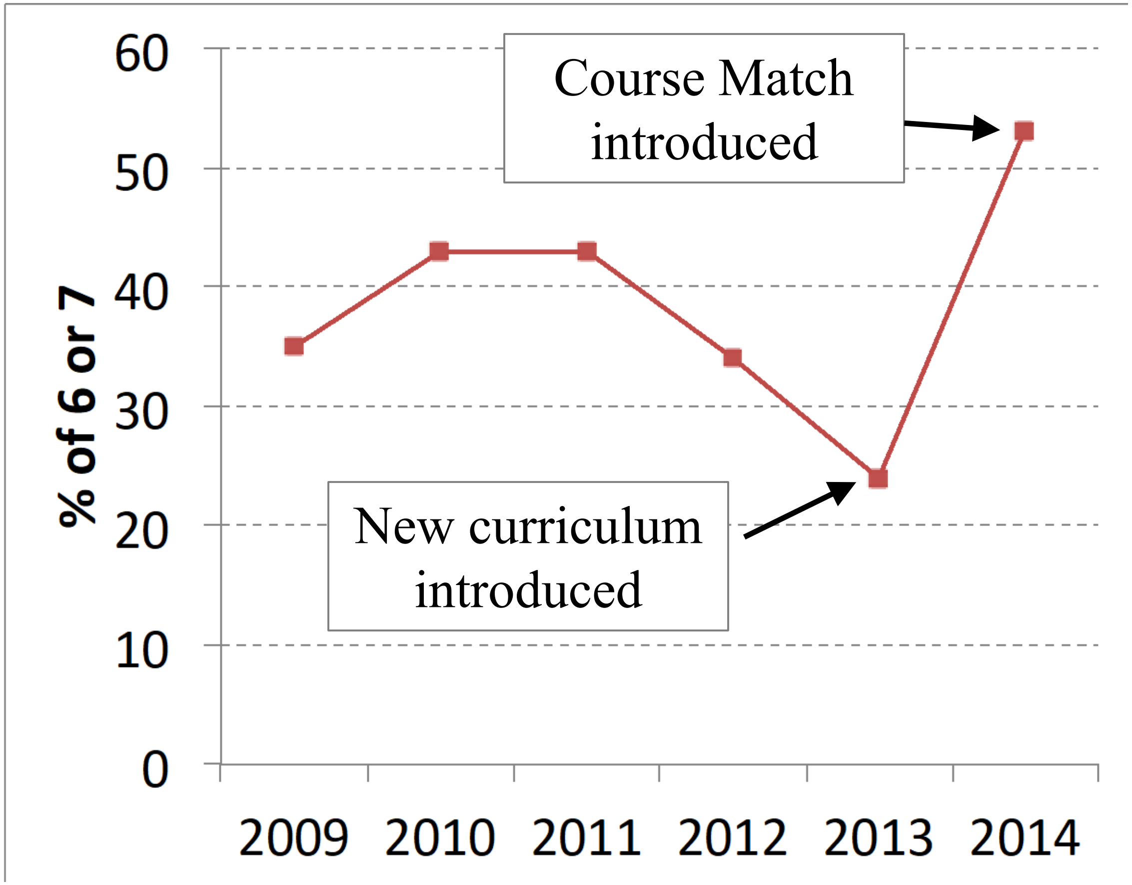


Fairness – Spring semester



Student survey response - effectiveness

"Please rate the effectiveness of the course auction/match system" (1-7 scale)



Student survey response – satisfaction & fairness

I was satisfied with my schedule from {the course auction system, course match}

	2013	2014	Change
Agree (6) + Strongly agree (7)	45	64	19
Somewhat Agree (5)	20	19	-1
Neither (4)	4	3	-1
Somewhat Disagree (3)	15	7	-8
Disagree (2) + Strongly disagree (1)	16	7	-9

{The course auction, Course match} allows for a fair allocation of classes.

	2013	2014	Change
Agree (6) + Strongly agree (7)	28	65	37
Somewhat Agree (5)	23	20	-3
Neither (4)	7	5	-2
Somewhat Disagree (3)	16	4	-12
Disagree (2) + Strongly disagree (1)	26	6	-20

Conclusion

- Existing theory requires modification to yield a feasible solution in practice

- Course Match substantially …
  - Reduced inequality among students
  - Improved student perception of effectiveness, satisfaction and fairness.

- Bonus:
  - Course Match provides reliable data on actual student preferences