Flow Trading

Eric Budish Peter Cramton Albert Kyle Mina Lee David Malec

WFA Portland
June 2022

Motivation: Trading Portfolios

- We propose a new market design for trading financial assets, which we call "flow trading"
- Trading portfolios
  - Going back to the CAPM and APT, optimal strategies involve trading portfolios of assets
  - Demand for one asset depends on the prices of other assets
- However, in the current financial market design, assets are traded in its own separate limit order book

Potential Factors

- Market clearing may be difficult to obtain
  - The existence of equilibrium requires goods to be generally substitutes or complements with restrictions
- Computation may not be feasible
  - Even if they exist, finding market clearing prices may be computationally intractable
- Path dependence
  - A human can run a limit order book market for individual assets with pen and paper or simple electronic recordkeeping

Flow Trading

- Participants can directly trade any user-defined portfolios
    - Any linear combinations with positive or negative weights
    - Markets clear asset by asset
- Markets clear in discrete time (Budish, Cramton and Shim (2015))
    - Especially important when all assets clear the market jointly
- Orders specify piecewise-linear downward-sloping demand schedules, with quantity expressed as flows (Kyle and Lee (2017))
    - Allow participants to trade gradually
    - Preferences are continuous in prices

Benefits

- From combining prior market design proposals,
  - BCS: addresses latency arbitrage and the arms race for trading speed
  - KL: gradual trading saves significant costs of the complex trading platform
  - BCS + KL: address inefficiencies related to the tick-size constraints
- From allowing direct trading for portfolios,
  - ETF trading now 40% of the stock market volume, charging 20 bps on average
  - The size of the industry built around fast arbitrage
- Improves transparency and fairness
- Caveat: does not mitigate market failures due to market power or private information

Preview of Technical Results

- Proves that the market clearing prices and quantities always exist, with unique market clearing quantities
  - Preference language is expressive enough to be useful but simple enough to guarantee existence
- Shows computational feasibility by providing a proof-of-concept based on simulated order books
  - Can clear markets with 500 assets and 100,000 orders in a baseline scenario in 0.15 seconds
- Provides a stylized microfoundation in the static CARA-normal framework
  - Optimal strategies in the theoretical framework can be implemented exactly in the flow trading market design

Flow Orders and Market Clearing

Flow Orders

- An order i is specified by a tuple $(\boldsymbol{w}^i, q^i, p_L^i, p_H^i, Q_i^{\text{max}})$
- The order is canceled once Q_i^{max} has been reached
- Flow portfolio demand is defined by

$$D^i\left(p^i := \boldsymbol{p}^\top \boldsymbol{w}^i\right) = \begin{cases} q^i, & \text{if } p^i \le p_L^i \\ q^i\left(\frac{p_H^i - p^i}{p_H^i - p_L^i}\right), & \text{if } p_L^i < p^i < p_H^i \\ 0, & \text{if } p^i \ge p_H^i \end{cases}$$

- Demand is continuous in prices without the explosion of message traffic

Market Clearing

- Market clears in assets but demands are for portfolios
- Convert portfolio units to underlying assets to calculate net excess demand vector for assets. Find $\boldsymbol{p}$ such that

$$\sum_{i=1}^{I} D^i \left(\boldsymbol{p}^\top \boldsymbol{w}^i\right) \cdot \boldsymbol{w}^i = \mathbf{0} \tag{1}$$

- Let x_i denote flow portfolio demand evaluated at a given portfolio price ($x_i := D^i \left(\boldsymbol{p}^\top \boldsymbol{w}^i\right)$)

Our Strategy

- First, define the pseudo marginal utility as the inverse of the demand schedule:

$$M^i(x_i) = p^i \tag{2}$$

where $x_i = D^i(p^i)$ and $p^i \in [p^i_L, p^i_H]$

- Then integrate the marginal utility to find

$$V^i(x_i) := \int_0^{x_i} M^i(u)du = p^i_H x_i - \frac{p^i_H - p^i_L}{2q^i} (x_i)^2 \,, \tag{3}$$

where $0 \leq x_i \leq q^i$ for all i.

Our Strategy

- Market clearing transforms into the quadratic programing problem of finding $\boldsymbol{x} = (x_1, \ldots, x_I)$ to solve

$$\max_{\boldsymbol{x}} \sum_{i=1}^{I} V^i(x_i) \qquad \text{subject to} \begin{cases} \sum_{i=0}^{I} x_i \boldsymbol{w}^i = \boldsymbol{0} \\ 0 \leq x_i \leq q^i \text{ for all } i \end{cases} \tag{4}$$

- Has nice properties to prove that there exists a unique quantity vector and there exist market clearing prices

- A sweet spot? Expressive enough to be useful, and existence is guaranteed

Computation

Computation

- We need to show that our proposal is computationally feasible
- The goal is to provide a proof of concept, clearing the market in less than a second for a reasonably difficult problem using an ordinary workstation
- We simulate an order book with 500 assets and 100,000 orders, and study robustness
- We use the interior point method, quadratic programming algorithm with superior computing performance

Exchange Trading

- We introduce (small amount of) exchange trading as "a market maker of last resort"
- The exchange submits a linear demand curve for each asset

$$\epsilon_n(\pi_{0n} - \pi_n), \tag{5}$$

where ϵ_n is the slope and π_{0n} is the vector of prices below which the exchange buys and above which it sells

- Makes prices unique and computation more efficient

Simulation Results

- On an ordinary workstation, basecase computation takes about 0.1451 seconds
  - Exchange trades 3.2 dollars per million; Uncleared quantities of 8.7 dollars per trillion
- Computation times range from 0.1159 to 0.2655 seconds when varying one parameter at a time, increase to 0.4291 seconds in the extreme scenario
- Computation times increase to 0.5639 and 4.67 seconds with 1,000,000 and 10,000,000 orders and 1.1021 and 56.3 seconds with 2,000 and 10,000 assets

Microfoundation

Static CARA-Normal Framework

- A trader finds the optimal portfolio that solves

$$\max_{\boldsymbol{\omega}} \mathrm{E}\left[-\exp^{-A(\boldsymbol{v}-\boldsymbol{\pi})^{\intercal}\boldsymbol{\omega}}\right], \tag{6}$$

  where $\boldsymbol{v}$ is the vector of asset payoffs.

- It is equivalent to the quadratic maximization problem, whose first order condition yields

$$\boldsymbol{\omega}^{*} = (A\boldsymbol{\Sigma})^{-1}(\mathbf{m}-\boldsymbol{\pi}), \tag{7}$$

  where $\boldsymbol{\Sigma}$ is the variance-covariance matrix and $\mathbf{m}$ is the vector of mean payoffs.

- The optimal demand for an asset generally depends on the prices of all assets

Singular Value Decomposition

- Since $\mathbf{\Sigma}$ is positive semidefinite, we have

$$\mathbf{\Sigma} = \mathbf{U}\mathbf{\Delta}\mathbf{U}^{\mathsf{T}}, \tag{8}$$

  where $\mathbf{U}$ is an orthonormal matrix, and $\mathbf{\Delta}$ is a diagonal matrix with nonnegative elements

- Using this, we can express the optimal demand as

$$\boldsymbol{\omega}^* = \sum_{i=1}^{K} \left(\frac{\mathbf{u}_i{}^{\mathsf{T}}\mathbf{m} - \mathbf{u}_i{}^{\mathsf{T}}\boldsymbol{\pi}}{A\delta_i} \right) \mathbf{u}_i, \tag{9}$$

  The demand for each portfolio only depends on the portfolio's price!

Generalization and Limitations

- With strategic trading, the same method still works when the price impact matrix is positive semidefinite (buying any portfolio increases the price of that portfolio)
- For any strictly concave, twice continuously differentiable quasi-linear preference, the optimal demand can be locally approximated with a combination of downward-sloping linear demand curves for portfolios
- May not work, however, with wealth effects or learning from prices due to upward-sloping demand curves

Conclusion

- Propose a new market design for trading financial assets by combining
  - Orders for portfolios
  - Frequent batch auctions
  - Piece-wise linear demand schedules in flows
- Orders for portfolios are rich enough for direct expression of many kinds of trading demands, yet simple enough for computing market clearing prices and quantities