Flow Trading

Eric Budish      Peter Cramton      Albert Kyle      Mina Lee      David Malec

WFA Portland
June 2022
Motivation: Trading Portfolios

- We propose a new market design for trading financial assets, which we call “flow trading”

- Trading portfolios
  - Going back to the CAPM and APT, optimal strategies involve trading portfolios of assets
  - Demand for one asset depends on the prices of other assets

- However, in the current financial market design, assets are traded in its own separate limit order book
Potential Factors

- Market clearing may be difficult to obtain
  - The existence of equilibrium requires goods to be generally substitutes or complements with restrictions

- Computation may not be feasible
  - Even if they exist, finding market clearing prices may be computationally intractable

- Path dependence
  - A human can run a limit order book market for individual assets with pen and paper or simple electronic recordkeeping
Flow Trading

- Participants can directly trade any user-defined portfolios
  - Any linear combinations with positive or negative weights
  - Markets clear asset by asset

- Markets clear in discrete time (Budish, Cramton and Shim (2015))
  - Especially important when all assets clear the market jointly

- Orders specify piecewise-linear downward-sloping demand schedules, with quantity expressed as flows (Kyle and Lee (2017))
  - Allow participants to trade gradually
  - Preferences are continuous in prices
Benefits

- From combining prior market design proposals,
  - BCS: addresses latency arbitrage and the arms race for trading speed
  - KL: gradual trading saves significant costs of the complex trading platform
  - BCS + KL: address inefficiencies related to the tick-size constraints

- From allowing direct trading for portfolios,
  - ETF trading now 40% of the stock market volume, charging 20 bps on average
  - The size of the industry built around fast arbitrage

- Improves transparency and fairness

- Caveat: does not mitigate market failures due to market power or private information
Preview of Technical Results

- Proves that the market clearing prices and quantities always exist, with unique market clearing quantities
  - Preference language is expressive enough to be useful but simple enough to guarantee existence

- Shows computational feasibility by providing a proof-of-concept based on simulated order books
  - Can clear markets with 500 assets and 100,000 orders in a baseline scenario in 0.15 seconds

- Provides a stylized microfoundation in the static CARA-normal framework
  - Optimal strategies in the theoretical framework can be implemented exactly in the flow trading market design
Flow Orders and Market Clearing
Flow Orders

- An order \( i \) is specified by a tuple \((w^i, q^i, p^i_L, p^i_H, Q^i_{\text{max}})\)

- The order is canceled once \( Q^i_{\text{max}} \) has been reached

- Flow portfolio demand is defined by

\[
D^i \left( p^i := p^\top w^i \right) = \begin{cases} 
q^i, & \text{if } p^i \leq p^i_L \\
q^i \left( \frac{p^i_H - p^i}{p^i_H - p^i_L} \right), & \text{if } p^i_L < p^i < p^i_H \\
0, & \text{if } p^i \geq p^i_H
\end{cases}
\]

- Demand is continuous in prices without the explosion of message traffic
Market Clearing

• Market clears in assets but demands are for portfolios

• Convert portfolio units to underlying assets to calculate net excess demand vector for assets. Find \( p \) such that

\[
\sum_{i=1}^{I} D^i \left( p^T w^i \right) \cdot w^i = 0
\] (1)

• Let \( x_i \) denote flow portfolio demand evaluated at a given portfolio price (\( x_i := D^i \left( p^T w^i \right) \))
Our Strategy

- First, define the pseudo marginal utility as the inverse of the demand schedule:

\[ M^i(x_i) = p^i \]  

(2)

where \( x_i = D^i(p^i) \) and \( p^i \in [p^i_L, p^i_H] \)

- Then integrate the marginal utility to find

\[ V^i(x_i) := \int_0^{x_i} M^i(u)du = p^i_Hx_i - \frac{p^i_H - p^i_L}{2q^i} (x_i)^2, \]  

(3)

where \( 0 \leq x_i \leq q^i \) for all \( i \).
Our Strategy

- Market clearing transforms into the quadratic programing problem of finding $x = (x_1, \ldots, x_I)$ to solve

$$\max_x \sum_{i=1}^I V^i(x_i) \quad \text{subject to} \quad \begin{cases} \sum_{i=0}^I x_iw^i = 0 \\ 0 \leq x_i \leq q^i \text{ for all } i \end{cases}$$

(4)

- Has nice properties to prove that there exists a unique quantity vector and there exist market clearing prices

- A sweet spot? Expressive enough to be useful, and existence is guaranteed
Computation
We need to show that our proposal is computationally feasible.

The goal is to provide a proof of concept, clearing the market in less than a second for a reasonably difficult problem using an ordinary workstation.

We simulate an order book with 500 assets and 100,000 orders, and study robustness.

We use the interior point method, quadratic programming algorithm with superior computing performance.
• We introduce (small amount of) exchange trading as “a market maker of last resort”

• The exchange submits a linear demand curve for each asset

\[ \epsilon_n (\pi_{0n} - \pi_n), \]  

(5)

where \( \epsilon_n \) is the slope and \( \pi_{0n} \) is the vector of prices below which the exchange buys and above which it sells

• Makes prices unique and computation more efficient
Simulation Results

- On an ordinary workstation, basecase computation takes about 0.1451 seconds
  - Exchange trades 3.2 dollars per million; Uncleared quantities of 8.7 dollars per trillion

- Computation times range from 0.1159 to 0.2655 seconds when varying one parameter at a time, increase to 0.4291 seconds in the extreme scenario

- Computation times increase to 0.5639 and 4.67 seconds with 1,000,000 and 10,000,000 orders and 1.1021 and 56.3 seconds with 2,000 and 10,000 assets
Microfoundation
● A trader finds the optimal portfolio that solves

$$\max_{\omega} \mathbb{E} \left[ -\exp^{-A(v-\pi)^T\omega} \right],$$  \hspace{1cm} (6)

where $v$ is the vector of asset payoffs.

● It is equivalent to the quadratic maximization problem, whose first order condition yields

$$\omega^* = (A \Sigma)^{-1}(m - \pi),$$  \hspace{1cm} (7)

where $\Sigma$ is the variance-covariance matrix and $m$ is the vector of mean payoffs.

● The optimal demand for an asset generally depends on the prices of all assets.
• Since $\Sigma$ is positive semidefinite, we have

$$\Sigma = U\Delta U^T,$$

where $U$ is an orthonormal matrix, and $\Delta$ is a diagonal matrix with nonnegative elements

• Using this, we can express the optimal demand as

$$\omega^* = \sum_{i=1}^{K} \left( \frac{u_i^T m - u_i^T \pi}{A\delta_i} \right) u_i,$$

The demand for each portfolio only depends on the portfolio’s price!
• With strategic trading, the same method still works when the price impact matrix is positive semidefinite (buying any portfolio increases the price of that portfolio)

• For any strictly concave, twice continuously differentiable quasi-linear preference, the optimal demand can be locally approximated with a combination of downward-sloping linear demand curves for portfolios

• May not work, however, with wealth effects or learning from prices due to upward-sloping demand curves
Propose a new market design for trading financial assets by combining
- Orders for portfolios
- Frequent batch auctions
- Piece-wise linear demand schedules in flows

Orders for portfolios are rich enough for direct expression of many kinds of trading demands, yet simple enough for computing market clearing prices and quantities.