A New Mechanism for Efficient and Fair Course Allocation: Data, Theory, Computation, Experiments, and Practical Implementation

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The Combinatorial Assignment Problem

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- Agents demand bundles of the objects
- Monetary transfers are exogenously prohibited
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Specific instance: Course Allocation at Universities

- Objects: seats in courses (scarce due to limits on class size)
- Agents: students, each of whom requires a schedule of courses
- Constraint against money: tuition does not vary based on which professors/classes the student takes (even at Chicago!)
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Other examples: assigning interchangeable workers to tasks or shifts; leads to salespeople; takeoff and landing slots to airlines; shared scientific resources amongst scientists; players to teams
Relation to the Literature

Combinatorial assignment is one feature removed from several canonical market design problems that have received considerable attention and have compelling solutions.
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- Divisible Goods → Classic Fair Division Problem
  - Theory: Steinhaus 1948 ...
Yet, Progress Has Been Elusive

Dictatorship Theorem. The only mechanisms that are ex-post Pareto efficient and strategy-proof are dictatorships (Klaus and Miyagawa, 2001; Papai, 2001; Ehlers and Klaus, 2003; Hatfield, 2009).

What is a dictatorship in this context?

Alice chooses her favorite bundle of courses.

Betty chooses her favorite bundle of courses, out of those not yet at capacity.

Zoe chooses her favorite bundle of courses, out of those not yet at capacity.

Other negative results for closely related problems (Sonmez, 1999; Konishi, Quint and Wako, 2001; Klaus and Miyagawa, 2001; Manea, 2007; Kojima, 2012).

Impossibility theorems are even more severe if we seek ex-ante Pareto efficiency (Zhou, 1990).
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- Papai (2001, p. 270): "[t]he implications are clear (...) if strategic manipulation is an issue, one should seriously consider using a serial dictatorship, however restrictive it may seem."

- Ehlers and Klaus (2003, p. 266): "[a] practical advantage of dictatorships is that they are simple and can be implemented easily. Furthermore, they are efficient, strategyproof (...). They can be considered to be ‘fair’ if the ordering of the agents is fairly determined; for instance by queuing, seniority, or randomization."

- Hatfield (2009, p. 514): "[the] results have shown that the only acceptable mechanisms for allocation problems of this sort is a sequential dictatorship, even when we restrict preferences to be responsive (...). Although unfortunate, it seems that in many of these applications, the best procedure (...) may well be a random serial dictatorship."
What should we make of this?

This conclusion is somewhat understandable:

▶ SP + ex-post Pareto efficiency are important properties in market design.
▶ SP + ex-post Pareto efficiency are central in the theory of single-unit assignment.
▶ Economists' tendency to view efficiency > fairness

But I think it's a flawed conclusion:
▶ Fairness seems an important objective in practice
▶ Wharton: goal is an "equitable and efficient allocation of seats in elective courses when demand exceeds supply"
▶ At very least, fairness should be viewed as an important constraint (akin to "repugnance"). Dictatorship level of unfairness a non-starter
▶ Wrong efficiency notion.
▶ What we care about is not ex-post Pareto efficiency, but ex-ante welfare
▶ Ex-post Pareto is very weak if not constrained by fairness considerations (e.g. agent 1 gets the whole endowment)
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My own takeaway: there is a basic tension among concerns of efficiency, fairness, and incentives. Any new market design will involve compromise of competing design objectives.
This Talk: Design of A New Mechanism

1. Given that theory is stuck, a sensible starting point is to see what we can learn from mechanisms that are actually used in practice. ▶ “The Multi-Unit Assignment Problem: Theory and Evidence from Course Allocation at Harvard” (w Cantillon, AER 2012)


4. Experimental test of the new mechanism. ▶ “Changing the Course Allocation Mechanism at Wharton” (w Judd Kessler, 2014 working paper)

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A Mechanism from Practice: the “Draft”

In practice we rarely observe dictatorships, in which agents take turns choosing their entire bundle of objects.

- Sports drafts (professional and playground)
- Allocation of tasks/shifts to workers
- Harvard Business School’s course draft
  1. Students submit preferences, in the form of an ROL over courses (implicit assumption: preferences are responsive)
  2. Students are randomly ordered by the computer
  3. Students are allocated courses one at a time, based on their reported preferences and remaining availability
  - Rounds 1, 3, 5, ...: ascending priority order
  - Rounds 2, 4, 6, ...: descending priority order
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It is easy to show that the draft is not strategy-proof (cf. Example 1 of BC 2012)

- Incentive to overreport "popular courses", underreport "unpopular courses"
- Intuition: don’t waste early round draft picks on courses that will sell out much later

It is also straightforward to show that the draft is not ex-post Pareto efficient in Nash equilibrium

Similar results in Brams and Straffin (1979), Manea (2007), for slightly different game forms

So, on the properties emphasized by Abdulkadiroglu and Sonmez (2003), Papai (2001), Ehlers and Klaus (2003), and Hatfield (2009):
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- Students’ actual submitted ROLs (potentially strategic)
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This combination of truthful and stated preferences is powerful for two reasons

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   - Does the lack of strategy-proofness actually matter in practice?

2. We can simulate equilibrium play of the (Random Serial) Dictatorship, and so can compare the two mechanisms
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Summary of Empirical Results

1. Students heavily manipulate the draft in practice
2. This misreporting harms efficiency, both ex-post and ex-ante
   ▶ Eqm misreporting is harmful, and in addition we find that students make optimization mistakes which causes further harm
3. Yet, the draft generates greater welfare than does the SP and ex-post efficient dictatorship:

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2. This misreporting harms efficiency, both ex-post and ex-ante
   - Eqm misreporting is harmful, and in addition we find that students make optimization mistakes which causes further harm
3. Yet, the draft generates greater welfare than does the SP and ex-post efficient dictatorship:

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HBS vs. RSD: Ex-Ante, Societal Level

Comparison of the societal average rank distribution under HBS-actual to RSD-truthful.
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Why is RSD so unattractive ex-ante? Example

Suppose there are 4 courses with capacity of 12 seats each. Students require 2 courses each. Preferences are as follows:

N2 students are P1: a, b, c, d
N2 students are P2: b, a, d, c

What happens under RSD?

Pr12: get 1st and 2nd favorites (a, b)
Pr12: get 3rd and 4th favorites (c, d)

What happens under HBS?

Always get 1st and 3rd favorites
P1 types always get {a, c}, P2 types get {b, d}

Note: truthful play is an eqm in this example
Why is RSD so unattractive ex-ante? Example

Suppose there are 4 courses with capacity of $\frac{1}{2}N$ seats each. Students require 2 courses each. Preferences are as follows:

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▶ So much so that the HBS lottery over inefficient allocations looks more attractive ex-ante than RSD.

▶ No efficiency-fairness tradeoff
▶ Ex-post efficiency need not even proxy for ex-ante efficiency
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- Seek a mechanism that yields a relatively equal distribution of outcomes, like the draft and unlike the dictatorship
  - Avoid dictatorship’s critical flaw – severe ex-post unfairness, which harms ex-ante welfare
Approximate CEEI

- Missing from both theory and practice is a mechanism that is attractive on all three dimensions of interest:
  - Efficiency
  - Fairness
  - Incentives

This paper proposes such a mechanism. Guided by lessons from Budish and Cantillon (2012). It gets around the impossibility theorems by making several small compromises versus the ideal properties a mechanism should satisfy.

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Definition. An allocation $x^*$, budget vector $b^*$ and price vector $p^*$ constitute an $(\alpha, \beta)$—approximate competitive equilibrium from equal incomes if:

1. Each student $i$ is allocated her most preferred bundle in her budget set: $x^*_i = \max\{u_i\}$ \quad \forall i \in S$
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**Theorem.** Let $k$ be the maximum number of courses in any permissible schedule. Define $\sigma = \min(2k, M)$. For any $\beta > 0$, there exists a $(\sqrt{\frac{\sigma M}{2}}, \beta)$-Approximate CEEI. 

Market-clearing error is at most $\sqrt{\sigma M}$, which is "small". Does not grow with market size as measured by $N$ or $q$ (as in Starr, 1969). Small number for practical problems, especially as a worst-case bound. Equal budgets ($\beta = 0$): market-clearing error could be arbitrarily large. Theorem tells us that "a little budget inequality goes a long way". Other extreme: dictatorships can be interpreted as exact CE ($\alpha = 0$), but from arbitrarily unequal budgets. Market administrator can assign these close but unequal budgets to agents however she likes (e.g., uniform randomly).
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  - Small number for practical problems, especially as a worst-case bound
- Equal budgets ($\beta = 0$): market-clearing error could be arbitrarily large
  - Theorem tells us that “a little budget inequality goes a long way”.
- Other extreme: dictatorships can be interpreted as exact CE ($\alpha = 0$), but from arbitrarily unequal budgets
Theorem 1: Existence of Approximate CEEI

**Theorem.** Let $k$ be the maximum number of courses in any permissible schedule. Define $\sigma = \min(2k, M)$. For any $\beta > 0$, there exists a $(\frac{\sqrt{\sigma M}}{2}, \beta)$-Approximate CEEI.

- Market-clearing error is at most $\frac{\sqrt{\sigma M}}{2}$, which is “small”
  - Does not grow with market size as measured by $N$ or $q$ (as in Starr, 1969)
  - Small number for practical problems, especially as a worst-case bound
- Equal budgets ($\beta = 0$): market-clearing error could be arbitrarily large
  - Theorem tells us that “a little budget inequality goes a long way”.
- Other extreme: dictatorships can be interpreted as exact CE ($\alpha = 0$), but from arbitrarily unequal budgets
- Market administrator can assign these close but unequal budgets to agents however she likes (e.g., uniform randomly)
Key Ideas in the Proof

- Basic difficulty: agents’ demands are discontinuous with respect to price, because of indivisibilities
  - Standard Arrow-Debreu-McKenzie existence results assume that preferences are continuous
- Role of $\sigma$: $\sqrt{\sigma}$ bounds the magnitude of an individual agent’s demand discontinuity
  - At worst, a small change in price can cause an agent’s demand to change from one bundle of objects to an entirely disjoint bundle of objects
- Role of unequal budgets: mitigate how individual agent demand discontinuities aggregate up into aggregate demand discontinuities
  - If agents have same budgets: discontinuities occur at the same points in price space. Can have “large” discontinuities: magnitude $N\sqrt{\sigma}$
  - If agents have distinct budgets: possible to change one agent’s choice set without changing all agents’ choice sets
  - Maximum magnitude of a discontinuity in aggregate demand is $M\sqrt{\sigma}$
Key Ideas in the Proof

- Can now apply an off-the-shelf approximate fixed point theorem (Cromme and Diener, 1991) to get market clearing error of at most $M\sqrt{\sigma}$
- Hard part of the proof is reducing the worst-case bound to $\frac{\sqrt{\sigma M}}{2}$
  - Exploits geometric structure of aggregate demand in a neighborhood of any $p$: zonotope
  - We can make an affordable / not-affordable decision for each agent-bundle pair where affordability near $p$ is in question
- Payoff to reducing the bound is meaningful for practice:
  - Semester at HBS: $M = 50$, $\sigma = 5$
  - $M\sqrt{\sigma} = 112$
  - $\frac{\sqrt{\sigma M}}{2} = 11$
A Simple Example: Two Diamonds, Two Rocks

Two agents. Four objects: two valuable Diamonds (Big, Small) and two ordinary Rocks (Pretty, Ugly). At most two objects per agent. Identical preferences.

Dictatorship?

Fairness problems: whoever's first gets both Diamonds.

CEEI?

Existence problems: at any price vector, for any object, either both agents demand it or neither does.

Approximate CEEI?

Randomly assign budgets of 1 and $1 + \beta$, for $\beta \geq 0$.

Set the price of the Big Diamond to be $1 + \beta$.

Set other prices such that the poorer agent can afford \{Small Diamond, Pretty Rock\}, wealthier agent gets \{Big Diamond, Ugly Rock\}.

Observe how budget inequality recovers existence (in this case exactly).
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  - Observe how budget inequality recovers existence (in this case exactly)
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“In fair division, the two most important tests of equity are ‘fair share guaranteed’ and ‘no envy’” (Moulin, 1995)
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  \[ u_i(x_i) \geq u_i(\frac{q}{N}) \text{ for all } i \]

- Envy freeness will be impossible to guarantee with indivisibilities. What if there is some single star professor whose course all students very badly want to take?
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Previous Approaches to Outcome Fairness with Indivisibilities

There have been several previous approaches to defining outcome fairness in the presence of indivisibilities:

1. Allow for monetary transfers (e.g., Alkan et al., 1991)
2. Assume that indivisible goods are actually divisible if needed (Brams and Taylor, 1999)
3. Assess criteria of outcome fairness at an interim stage (Hylland and Zeckhauser, 1979; Bogomolnaia and Moulin, 2001; Pratt, 2007)

Common thread in previous approaches:
▶ Modify either the problem or the time at which fairness is assessed
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Maximin Share Guarantee

Definition. Agent $i$’s **maximin share** is the consumption bundle

$$\max\{\min\{x_1, \ldots, x_N\}\}$$

where the $\max(\cdot)$ is taken over all feasible allocations. Any allocation in which all $N$ agents get a bundle they prefer to their maximin share is said to satisfy the **maximin share guarantee**

- Divide-and-choose interpretation
- Rawlsian guarantee from what Moulin (1991) calls a “thin veil of ignorance”
- Coincides with fair share if goods divisible, prefs convex and monotonic
Envy Bounded by a Single Good

Definition. An allocation $x$ satisfies **envy bounded by a single good** if, for any two agents $i, i'$, either:

1. $u_i(x_i) \geq u_i(x_{i'})$
2. $u_i(x_i) \geq u_i(x_{i'} \setminus \{j\})$ for some $j \in x_{i'}$

- In words: if student $i$ envies student $i'$ the envy is bounded: by removing some single good from $i'$’s bundle we would eliminate $i$’s envy

- Coincides with envy-freeness in a limit as consumption bundles become perfectly divisible
In diamonds and rocks example, Approximate CEEI satisfies both fairness criteria.
Diamonds and Rocks Revisited

- In diamonds and rocks example, Approximate CEEI satisfies both fairness criteria
  - Each agent’s maximin share is \{Small Diamond, Pretty Rock\}.

- Dictatorships satisfy neither criteria
  - Whichever agent goes first gets both diamonds
  - By contrast, in single-unit assignment problems (e.g., one diamond, one rock), dictatorships actually do satisfy both criteria
  - Dictatorships are frequently used in practice for single-unit assignment problems (school choice, housing assignment)

The fairness properties help us to make sense of the empirical patterns of dictatorship usage. Useful external validity check.
In diamonds and rocks example, Approximate CEEI satisfies both fairness criteria

- Each agent’s maximin share is \{Small Diamond, Pretty Rock\}.
- The agent who gets \{Small Diamond, Pretty Rock\} envies the agent who gets \{Big Diamond, Ugly Rock\}, but the envy is bounded by a single good (the big diamond).
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Theorem 2: If $\beta < \frac{1}{N}$ then $x^*$ guarantees each agent their $N + 1$-maximin share (maximin share in a hypothetical economy with one additional agent).

Intuition for proof:

1. If $\beta < \frac{1}{N}$ then even poorest student has $> \frac{1}{N+1}$ of the income endowment

2. If $p^*$ is an exact competitive equilibrium, then the goods endowment costs weakly less than the income endowment

3. So, if $p^*$ is an exact c.e., each student can afford some bundle in any $N+1$-way split

4. Hence, each student must be able to afford some bundle weakly preferred to her $N+1$-maximin share

The full argument is a bit messier because $p^*$ might be an approximate c.e.
Theorem 3: If $\beta < \frac{1}{k-1}$ then $x^*$ satisfies envy bounded by a single good.

Intuition for proof:

1. Suppose $i$ envies $j$. Then

$$1 \leq b_i^* < p^* \cdot x_j^* \leq b_j^* \leq \frac{k}{k-1}$$

2. Since $x_j^*$ contains at most $k$ goods, one of them must cost at least $\frac{1}{k-1}$. $i$ can afford the bundle formed by removing this good from $x_j^*$

3. Hence, $i$ must weakly prefer her own assigned bundle to the bundle formed by removing this single good from $x_j^*$, so her envy is bounded.
The Approximate CEEI Mechanism

1. Students report their preferences
The Approximate CEEI Mechanism

1. Students report their preferences
2. Students are given approximately equal budgets of an artificial currency – uniform-random draws from $[1, 1 + \beta]$ with $\beta$ suitably small
3. Compute an item price vector $p^*$ and allocations, in an anonymous manner, such that when each student $i$ is allocated her favorite bundle in her budget set $x^*_i = \max(u_i)$ \{$x \in 2^C : p^* \cdot x \leq b^*_i$}\, the market approximately clears (market-clearing error as small as possible, and certainly no larger than $\sqrt{\sigma M}$)
4. Allocate each student her demand at $p^*$ based on her reported preferences
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- Approximate CEEI is not strategy-proof in finite markets
- Instead, it is strategy-proof in a large-market sense – if students ignore their own influence on prices
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Formally, it is strategy-proof in the large (Azevedo and Budish, 2013)

- Other mechanisms that are not SP but are SP-L: Gale-Shapley deferred acceptance, uniform-price auctions, Walrasian mechanism, probabilistic serial
- Mechanisms that are not SP-L: Boston mechanism, priority-match algorithm, pay-as-bid auctions, HBS draft
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Properties of the Approximate CEEI Mechanism

Efficiency
*Ex-post efficient, but for small error*

Fairness
*N+1 Maximin Share Guarantee
Envy Bounded by a Single Good*

Incentives
*Strategy-proof in the Large*
TABLE 1
COMPARISON OF ALTERNATIVE MECHANISMS

<table>
<thead>
<tr>
<th>Mechanism</th>
<th>Efficiency (Truthful Play)</th>
<th>Fairness (Truthful Play)</th>
<th>Incentives</th>
<th>Preference Language</th>
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<tr>
<td>A-CEEI mechanism</td>
<td>Pareto efficient with respect to allocated goods</td>
<td>$N + 1$—maximin share guaranteed</td>
<td>Strategyproof in the large</td>
<td>Ordinal over schedules</td>
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<tr>
<td>Mechanisms from practice:</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Bidding points auction</td>
<td>If preferences are additive separable, Pareto efficient but for quota issues</td>
<td>Worst case: get zero goods</td>
<td>Manipulable in the large</td>
<td>Cardinal over items</td>
</tr>
<tr>
<td>(Sonmez and Unver 2003)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>HBS draft mechanism (Budish and Cantillon, forthcoming)</td>
<td>If preferences are responsive, Pareto efficient with respect to the reported information</td>
<td>If preferences are responsive and $k = 2$, maximin share guaranteed</td>
<td>Manipulable in the large</td>
<td>Ordinal over items</td>
</tr>
</tbody>
</table>
| Univ. Chicago primal-dual linear program mechanism (Graves, Schrage, and Sankaran 1993) | Pareto efficient when preference-reporting limits do not bind   | Worst case: get zero goods                                   | Manipulable in the large            | Cardinal over a limited num-
<p>| Mechanisms from prior theory:              |                                                                  |                                                                  |                                      | ber of schedules             |
| Adjusted winner (Brams and Taylor 1996)    | If preferences are additive-separable, Pareto efficient         | Worst case: get zero goods                                   | Manipulable in the large            | Cardinal over items          |</p>
<table>
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<tr>
<th>Mechanism</th>
<th>Efficiency</th>
<th>Preferences</th>
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<th>Cardinality</th>
</tr>
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<tr>
<td>Descending demand procedure (Herreiner and Puppe 2002)</td>
<td>Pareto efficient</td>
<td>Does not satisfy maximin share guarantee or envy bounded by a single good</td>
<td>Worst case: get zero goods</td>
<td>Manipulable in the large</td>
<td>Ordinal over schedules</td>
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<td>Gale-Shapley enhancement to the BPA (Sönmez and Unver 2003)</td>
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<td></td>
<td>Worst case: get zero goods</td>
<td>Bidding phase: manipulable in the large</td>
<td>Bidding phase: cardinal over items</td>
</tr>
<tr>
<td>Geometric prices mechanism (Pratt 2007)</td>
<td>If von Neumann–Morgenstern preferences are additive separable, Pareto efficient</td>
<td>Worst case: get zero goods</td>
<td></td>
<td>Allocation phase: strategyproof in the large</td>
<td>Allocation phase: ordinal over items</td>
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<tr>
<td>Minimize envy algorithm (Lipton et al. 2004)</td>
<td>Algorithm ignores efficiency</td>
<td>If preferences are additive separable, envy bounded by a single good</td>
<td>Manipulable in the large</td>
<td>Cardinal over schedules</td>
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<td>Serial/sequential dictatorship (cf. Pápai 2001)</td>
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<td>Worst case: get ( k ) worst goods</td>
<td>Strategyproof</td>
<td>Ordinal over schedules</td>
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<tr>
<td>Other solution concepts: Maximin utility</td>
<td>Pareto efficient</td>
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<td>Utilitarian solution</td>
<td>Pareto efficient</td>
<td>Worst case: get zero goods (if a depressive and all other agents are hedonists)</td>
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Othman, Sandholm and Budish: “Finding Approximate Competitive Equilibria: Efficient and Fair Course Allocation” (AAMAS, 2010)
Computational Procedure for Approximate CEEI

- Theorem 1 of Budish (2011) shows Approx CEEI prices exist
- Does not show how to find them
  - Proof is non-constructive
  - Approximate Kakutani fixed point
Computational Procedure for Approximate CEEI

- Othman, Sandholm and Budish (2010) develops a computational procedure for finding approximate market-clearing prices.

- Two level procedure:

1. Agent Level: computing students’ demands at a candidate price vector $p$
   - NP Hard: knapsack problem
   - But: doable in practical sized problems, and highly parallelizable (Hayek)

2. Price Level: searching through price space for approximate market-clearing prices
   - We use a method called “Tabu Search”
   - Departure point is the Tatonnement process:
     $p^{t+1} = p^t + z(p^t)$
   - Note: no guarantees ... open question whether there is a better method
Performance of Approximate CEEI on the HBS Data

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   ▶ That is, even a planner who does not care about fairness per se, only welfare, prefers Approximate CEEI